Interference of Two Beams:1

After passing through the two separate arms of the interferometer we have two beams with amplitudes

$$\begin{aligned} A_1 &= \cos(kx_1 - kct) \\ A_2 &= \cos(kx_2 - kct) \end{aligned}$$

where $k = \frac{2\pi}{\lambda}$ is the wavenumber, x_1 and x_2 are the optical path lengths in the two sides of the interferometer, t is the time and c is the speed of light. We mix the two signals and detect the time averaged intensity:

$$\begin{aligned} I &= \overline{(A_1 + A_2)^2} \\ &= \overline{\cos^2(kx_1 - kct)} + \overline{2\cos(kx_1 - kct)\cos(kx_2 - kct)} + \overline{\cos^2(kx_2 - kct)} \\ &= 1 + 2\overline{[\cos(kx_1)\cos(kct) + sin(kx_1)sin(kct)][\cos(kx_2)\cos(kct) + sin(kx_2)sin(kct)]} \\ &= 1 + \cos(kx_1)\cos(kx_2) + sin(kx_1)sin(kx_2) \\ &= 1 + \cos(k(x_1 - x_2)) \end{aligned}$$

If we define the optical path length difference to be $\Delta\mathrm{OPD} = x_1 - x_2$ and add the visibility function for the object $V(k)$ we get, at a single wavelength,

$$I(\nu) = 1 + V(\nu)\cos(2\pi\nu\,\Delta\mathrm{OPD})$$

where we now use the wavenumber $\nu = \frac{1}{\lambda}$.

Interference of Two Beams:2

In a real detector there will be a finite bandpass, say from $\nu_0 - \frac{\Delta\nu}{2}$ to $\nu_0 + \frac{\Delta\nu}{2}$ and so we have to integrate over this (assumed flat) bandpass:

$$
\begin{aligned}
I(\nu_0, \Delta\nu) &= \frac{1}{\Delta\nu}\int_{\nu_0-\frac{\Delta\nu}{2}}^{\nu_0+\frac{\Delta\nu}{2}} (1 + V(\nu)\cos[2\pi\nu\,\Delta\mathrm{OPD}])\,\mathrm{d}\nu \\
&\simeq 1 + \frac{1}{\Delta\nu}V(\nu_0)\int_{\nu_0-\frac{\Delta\nu}{2}}^{\nu_0+\frac{\Delta\nu}{2}} \cos[2\pi\nu\,\Delta\mathrm{OPD}]\,\mathrm{d}\nu
\end{aligned}
$$

We make the substitution

$$
\eta = \nu - \nu_0
$$

and the integral becomes

$$
\begin{aligned}
\mathcal{I} &= \int_{\nu_0-\frac{\Delta\nu}{2}}^{\nu_0+\frac{\Delta\nu}{2}} \cos[2\pi\nu\,\Delta\mathrm{OPD}]\,\mathrm{d}\nu \\
&= \int_{-\frac{\Delta\nu}{2}}^{+\frac{\Delta\nu}{2}} \cos[2\pi\,\Delta\mathrm{OPD}(\nu_0+\eta)]\,\mathrm{d}\nu \\
&= \frac{1}{2\pi\,\Delta\mathrm{OPD}}\left[\sin\left(2\pi\,\Delta\mathrm{OPD}\left[\nu_0+\frac{\Delta\nu}{2}\right]\right) - \sin\left(2\pi\,\Delta\mathrm{OPD}\left[\nu_0-\frac{\Delta\nu}{2}\right]\right)\right] \\
&= \frac{1}{2\pi\,\Delta\mathrm{OPD}}\left[2\cos(2\pi\nu_0\,\Delta\mathrm{OPD})\sin\left(2\pi\,\Delta\mathrm{OPD}\frac{\Delta\nu}{2}\right)\right]
\end{aligned}
$$

thus the detected signal will be

$$
I(\nu_0, \Delta\nu) = 1 + V(\nu_0)\frac{\sin(\pi\,\Delta\mathrm{OPD}\,\Delta\nu)}{\pi\,\Delta\mathrm{OPD}\,\Delta\nu}\cos(2\pi\nu_0\,\Delta\mathrm{OPD})
$$

This has a maximum when $\Delta\mathrm{OPD} = 0$ and has a first set of zeros at

$$
\pi\,\Delta\mathrm{OPD}\,\Delta\nu = \pm\pi
$$

$$
\Delta\mathrm{OPD} = \frac{\pm 1}{\Delta\nu}
$$

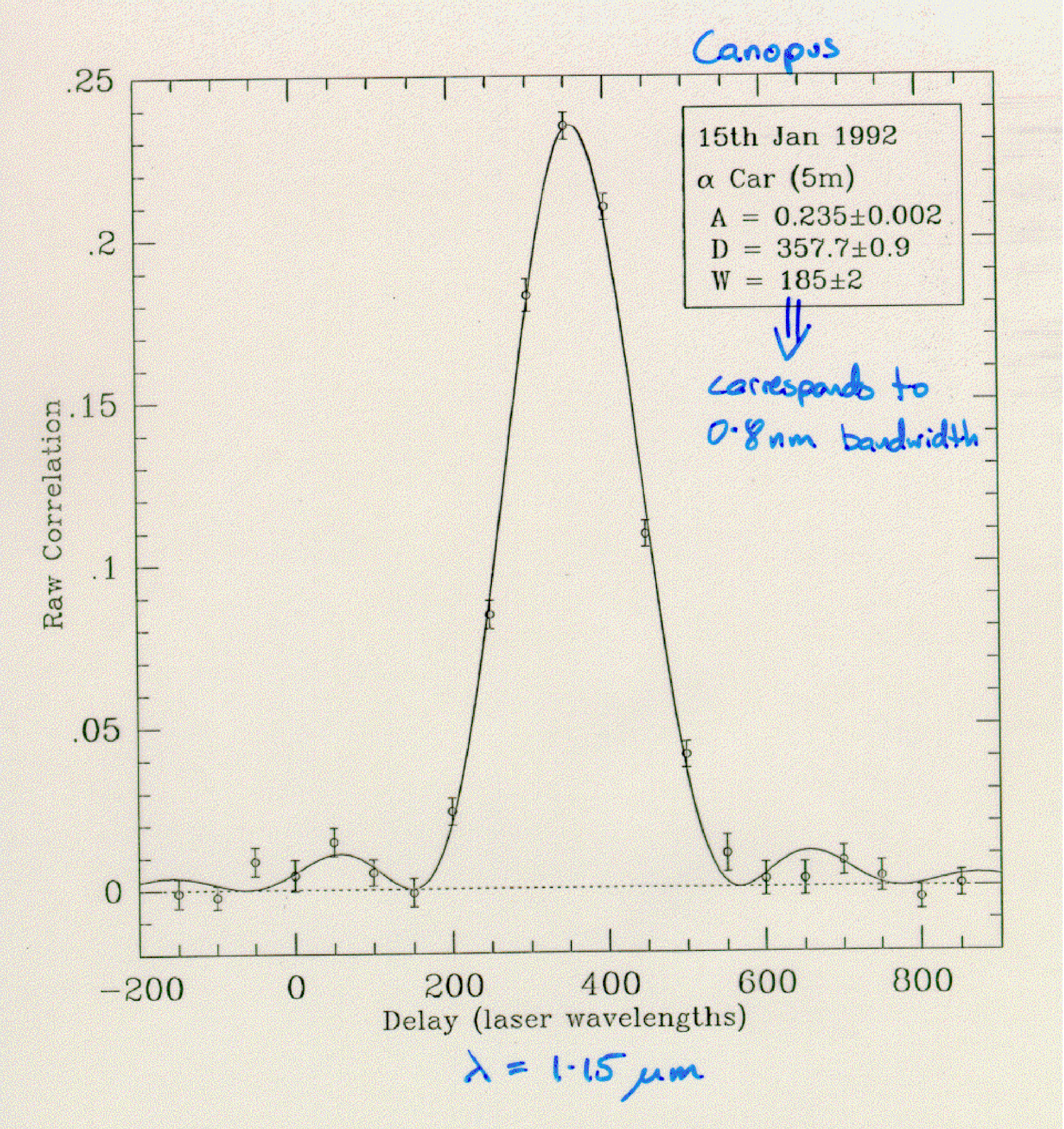
which means that the fringe coherence length will be $\frac{2}{\Delta\nu}$.

Fringe Equation

$$I(t,\nu) = 1 + V(t,\nu) \times \frac{\sin(\pi\ x(t,\nu)\ \Delta\nu)}{\pi\ x(t,\nu)\ \Delta\nu} \times \cos\left(2\pi\, x(t,\nu)\ \nu + \Phi_{\mathrm{obj}} + \Phi_{\mathrm{atm}}(t)\right)$$

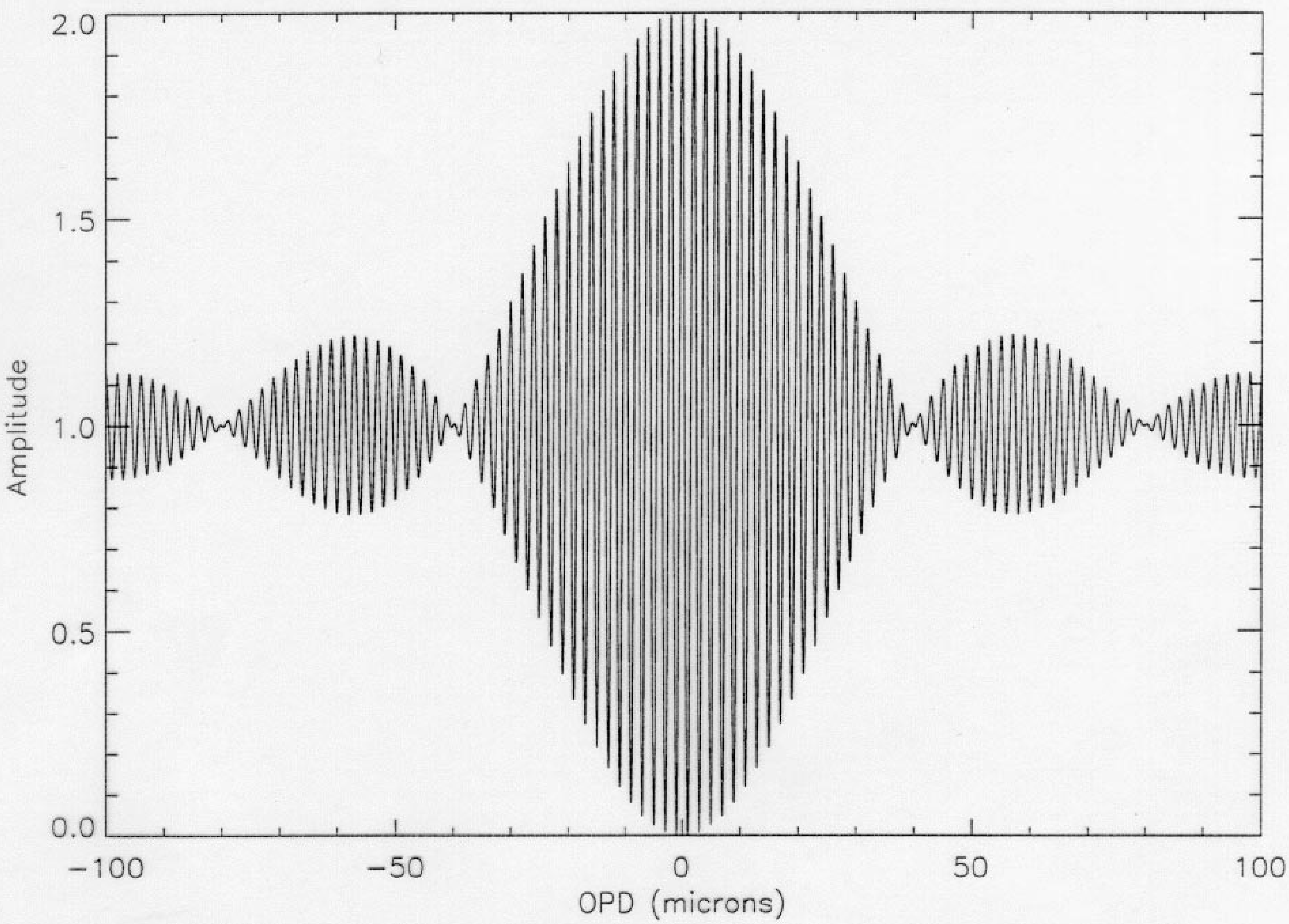
$$V(t,\nu) = V_{\mathrm{obj}} \times \eta_{\mathrm{atm}}(t) \times \eta_{\mathrm{inst}}$$

$$x(t,\nu) = B\cos\left(\theta(t)\right) + x_{\mathrm{vac}}(t) + n_{\mathrm{air}}(\nu) x_{\mathrm{air}}(t) + n_{\mathrm{glass}}(\nu) x_{\mathrm{glass}}(t)$$



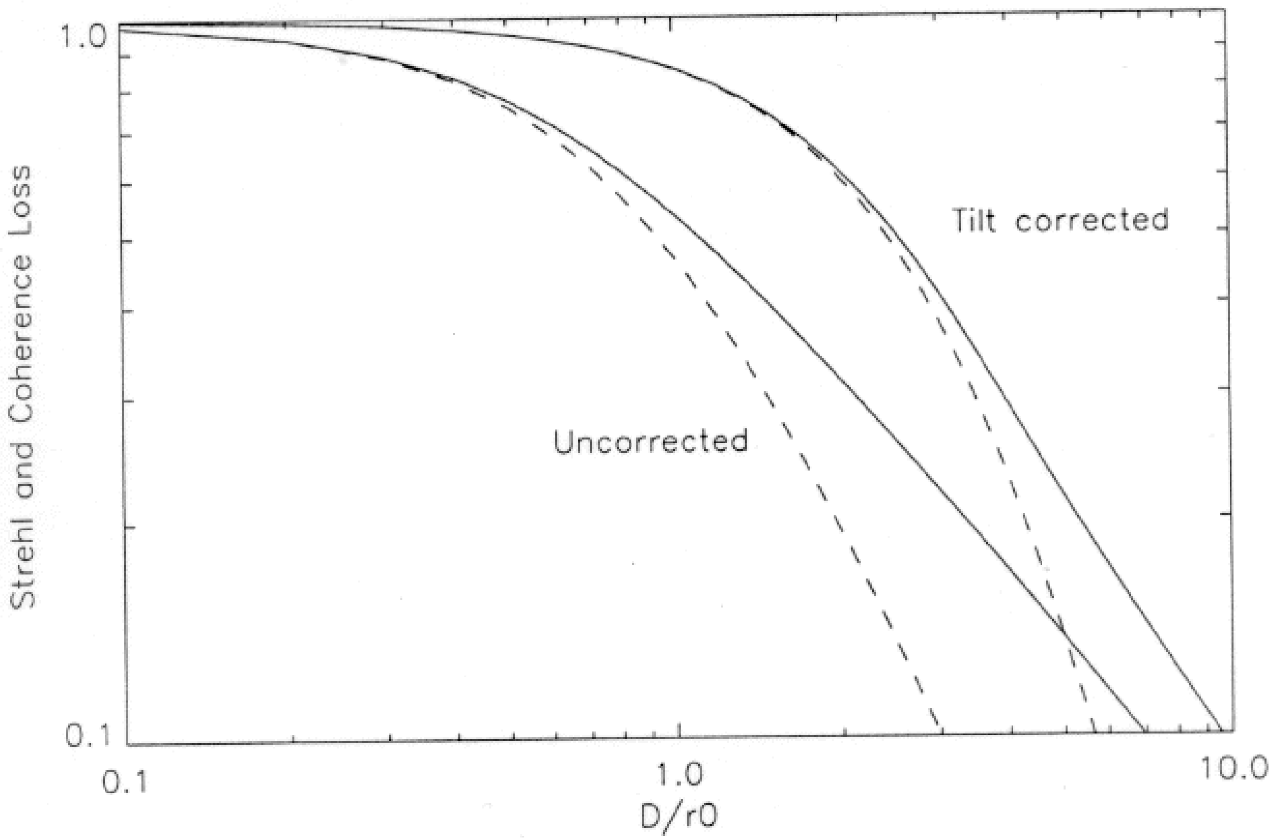
Canopus
15th Jan 1992
α Car (5m)
A = 0.235±0.002
D = 357.7±0.9
W = 185±2
corresponds to
0·8 nm bandwidth
Raw Correlation
.25
.2
.15
.1
.05
0
−200
0
200
400
600
800
Delay (laser wavelengths)
λ = 1·15 μm

Center Wavelength = 2.0 micron
Bandwidth = 0.05 micron

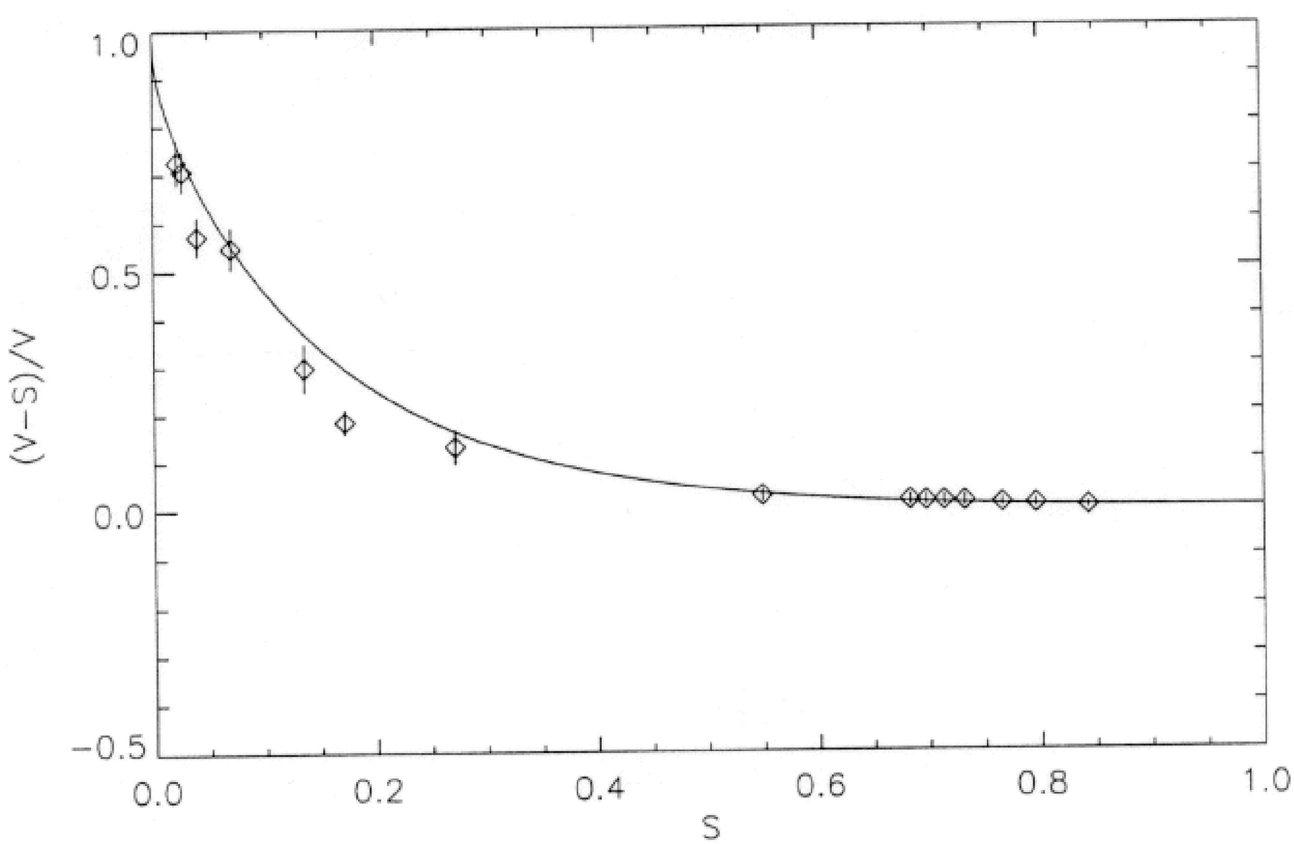


Things to worry about:

- Vibration
- Thermal stability
- Throughput
- Refraction (Angular)
- Polarization
- Dispersion (Linear)
- Diffraction
- Tip/tilt
- Higher order wavefront errors
- Optical path length equalization (OPLE)
- Logistics
- Fringe tracking
- Imaging (IR/Vis)



Strehl and Coherence Loss
Tilt corrected
Uncorrected
D/r0
1.0
0.1
0.1
1.0
10.0



(V−S)/V
S
1.0
0.5
0.0
−0.5
0.0
0.2
0.4
0.6
0.8
1.0

Mon. Not. R. Astron. Soc. 273, L53–L58 (1995)

Atmospheric path variations for baselines up to 80 m measured with the Sydney University Stellar Interferometer

J. Davis,[1] P. R. Lawson,[2] A. J. Booth,[1] W. J. Tango[1] and E. D. Thorvaldson[1]

[1] *Chatterton Astronomy Department, School of Physics, University of Sydney, NSW 2006, Australia*

[2] *Observatoire de la Côte d'Azur, URA 1361 de CNRS, Caussols, St Vallier de Thiey, F-06460, France*

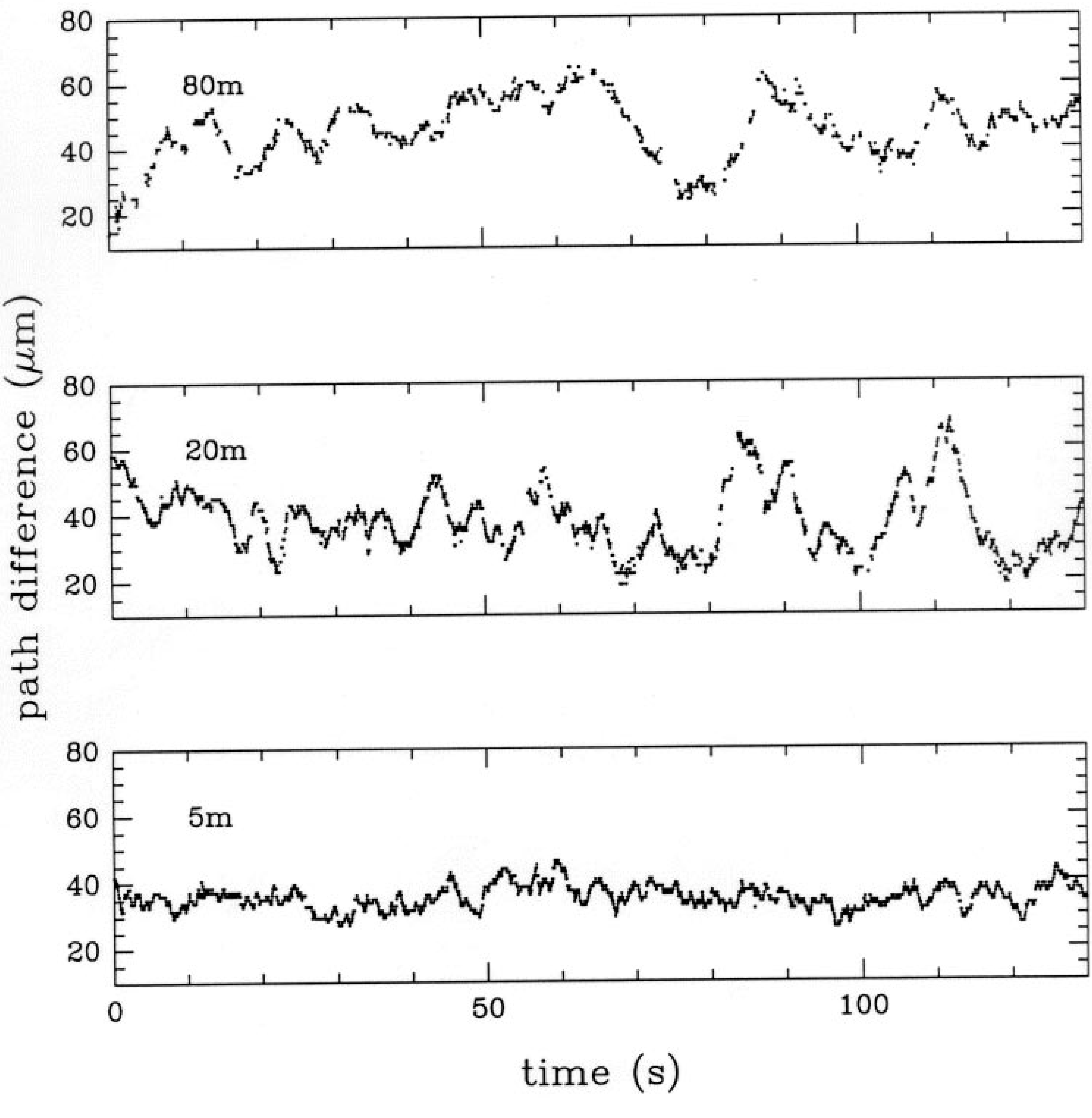


Table 1. Standard deviation of optical path length fluctuations [$\sigma_{opd}(x)$] averaged from 50-s long data sets for three baselines.

Baseline (m)	$\sigma_{opd}(x)$ (μm)	Source	Number of data sets
5	4.6 ± 0.6	α CMa	24
20	8.4 ± 0.6	α Vir	10
80	11.4 ± 1.8	α Vir	9

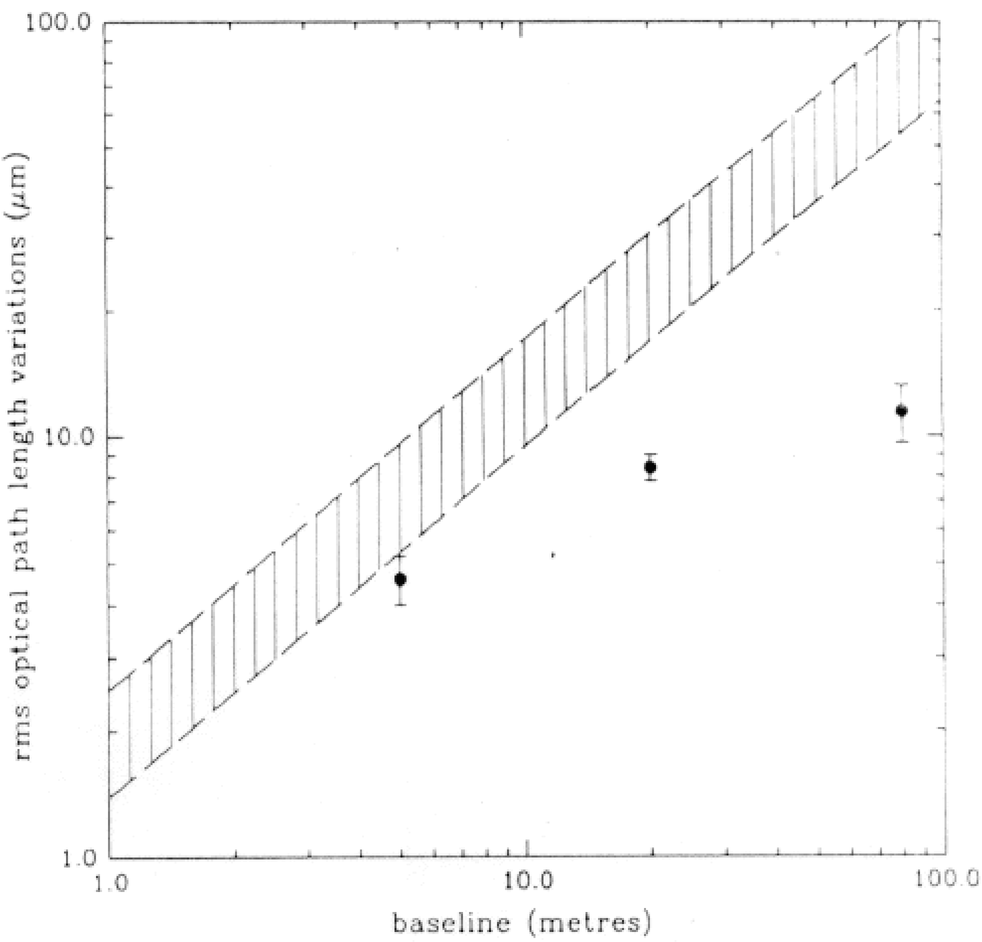


Figure 3. Root mean square optical path length fluctuations as a function of baseline. The three points are the measurements reported in this paper. The broken lines have the slope predicted for Kolmogorov turbulence with infinite outer scale, and represent the relationships for the extreme values of r_0 observed during the measurements of optical path variation. If the rms optical path variations were consistent with Kolmogorov statistics, the observed points would lie in the shaded area between the curves.

2.3. The Effect of Sampling Time

Based on the definition of coherence time given by Equation (1) it follows (Buscher, 1988) that, for small apertures (diameter $< r_0$), the time average of the correlation $\overline{C(\Delta t)}$ is given by

$$\frac{\overline{C(\Delta t)}}{C(0)} = \frac{2}{\Delta t}\int_0^{\Delta t}\left(1 - \frac{t}{\Delta t}\right) B(t)dt, \tag{2}$$

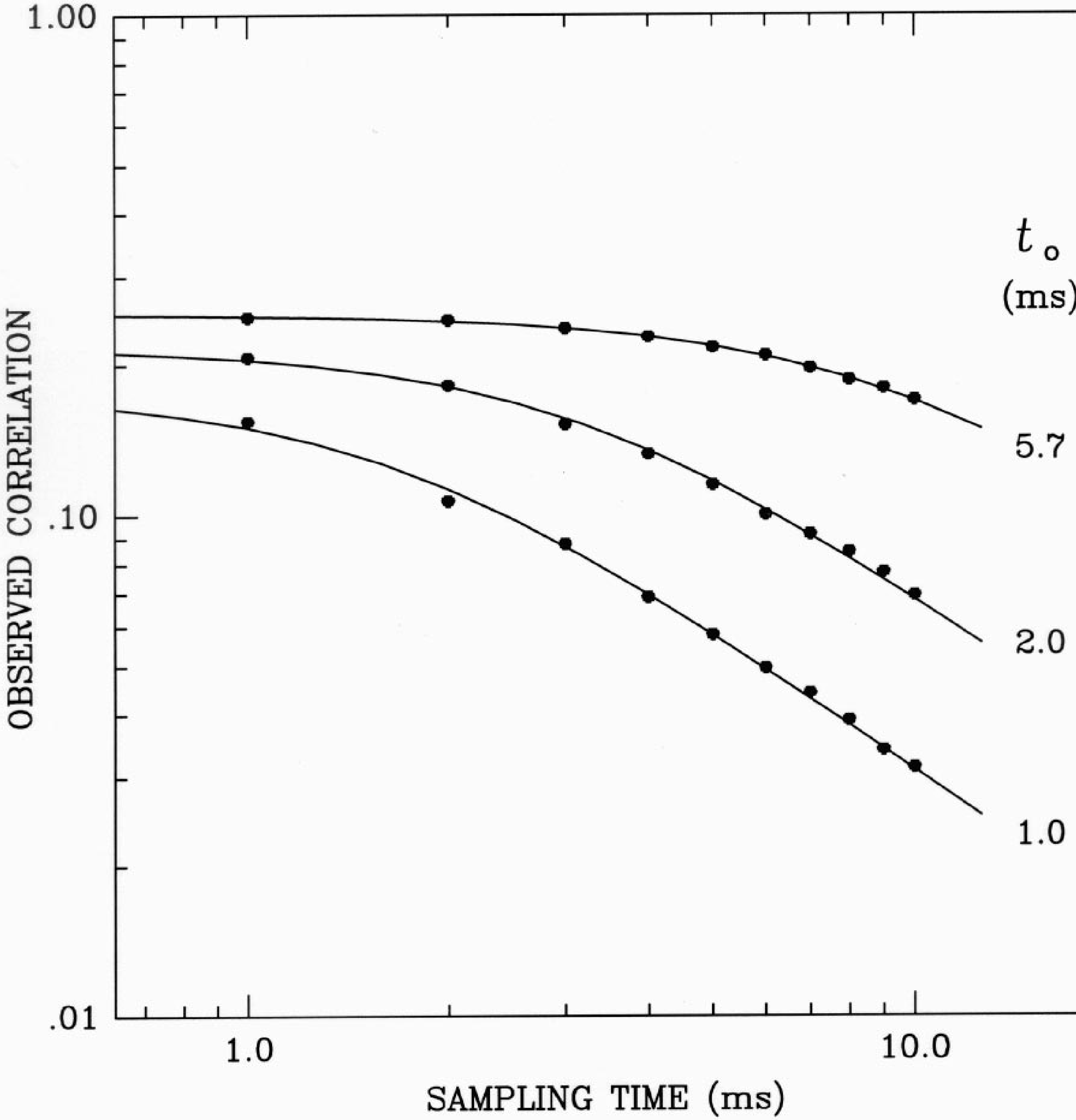
where $C(0)$ is the correlation that would be measured with a sample time $\Delta t \ll t_0$.

It can be shown that the analytic expression for Equation (2) is

$$\frac{\overline{C(\Delta t)}}{C(0)} = \frac{6}{5\tau}\left[\gamma(3/5, \tau^{5/3}) - \tau^{-1}\gamma(6/5, \tau^{5/3}\right], \tag{3}$$

where $\tau = \Delta t/t_0$ and $\gamma(a, x)$ is a partial gamma function.

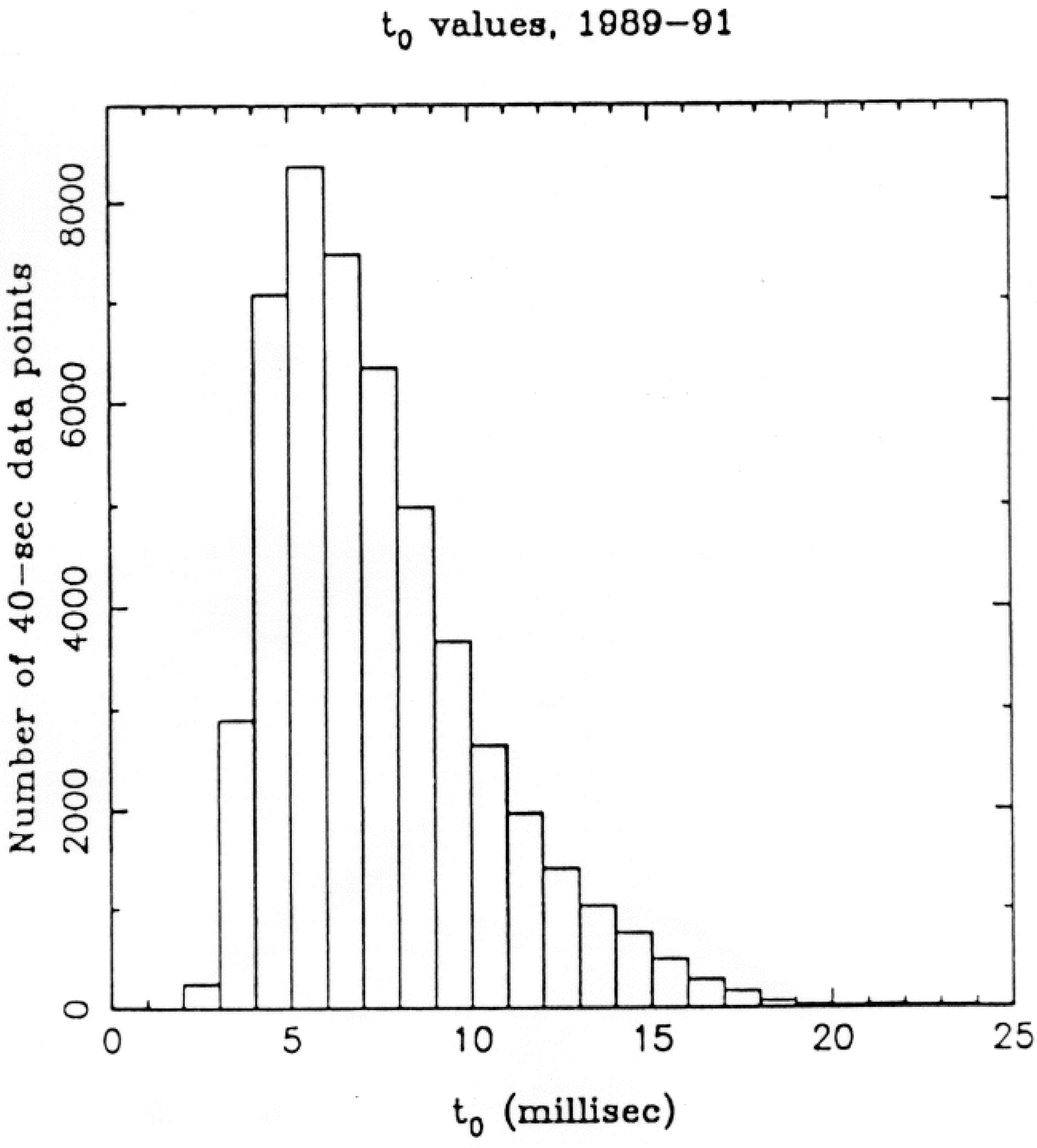
The ratio $\overline{C(\Delta t)}/C(0)$, which we call the "correlation loss factor", is shown as Figure 1 in Buscher (1988). It is plotted against sample time for a range of values of the coherence time in Fig. 1.



1.00
.10
.01
OBSERVED CORRELATION
1.0
10.0
SAMPLING TIME (ms)
t_o
(ms)
5.7
2.0
1.0

A Thousand and One Nights of Seeing on Mt Wilson

D.F. Buscher
Universities Space Research Association
and
NRL/USNO Optical Interferometer Project
U.S. Naval Observatory, AD5
3450 Massachusetts Ave NW, Washington DC 20392-5420
Email: dbuscher@atlas.usno.navy.mil



Polarization

- Each reflection introduces a polarization change.
- Each polarization state ‘sees’ a different phase shift on reflections.
- Common strategies for avoiding these problems:
  1. Use a completely (or almost completely) symmetric optical layout.
  2. Separate polarizations in the beam combiner.
- This is something you can (hopefully) measure in order to calibrate it out.

NOAO - ESO Conf 1988 Ed F Merkle

POLARIZATION EFFECTS IN STELLAR INTERFEROMETERS

Wesley A. Traub
Harvard-Smithsonian Center for Astrophysics
60 Garden Street
Cambridge, MA 02138

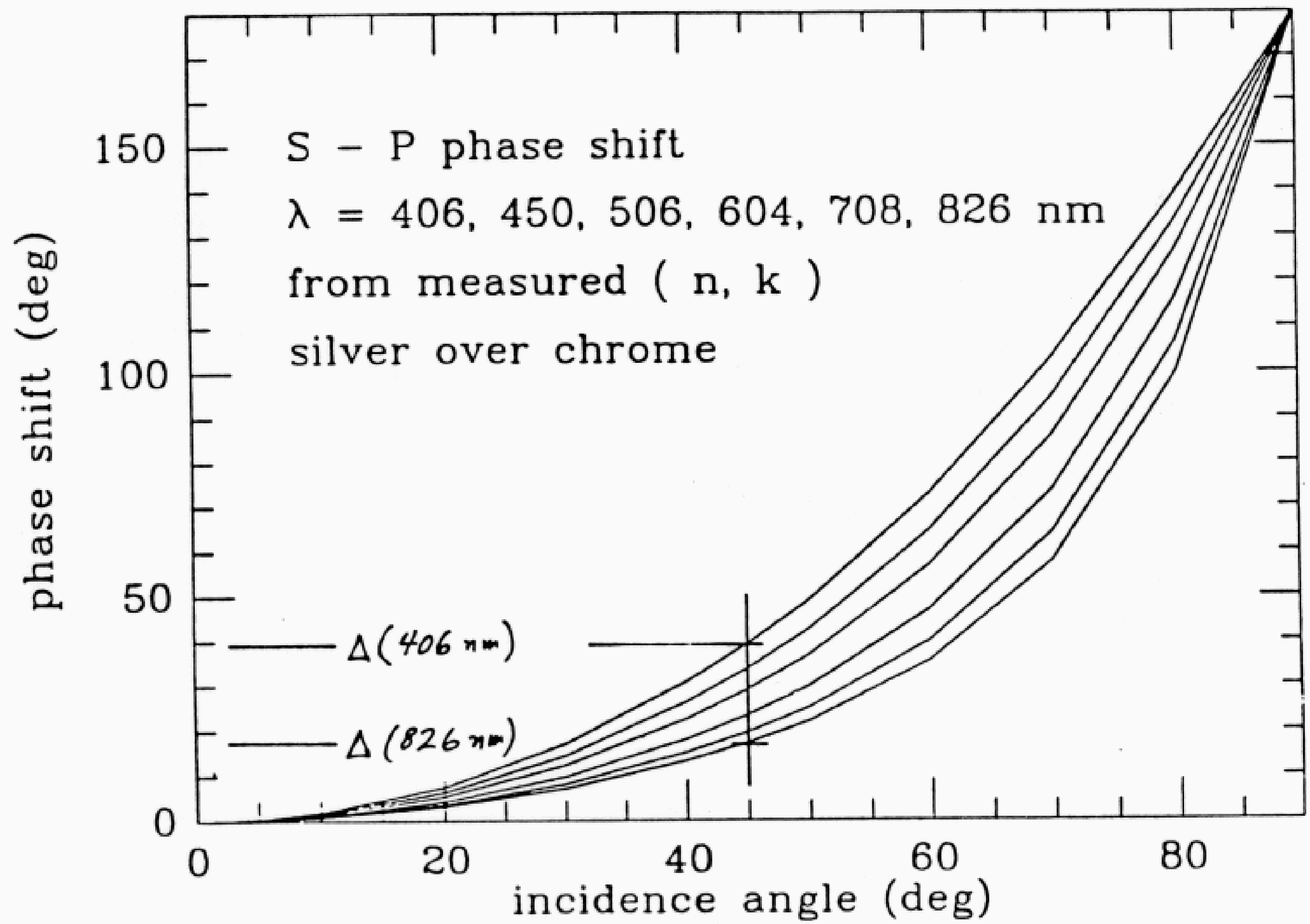
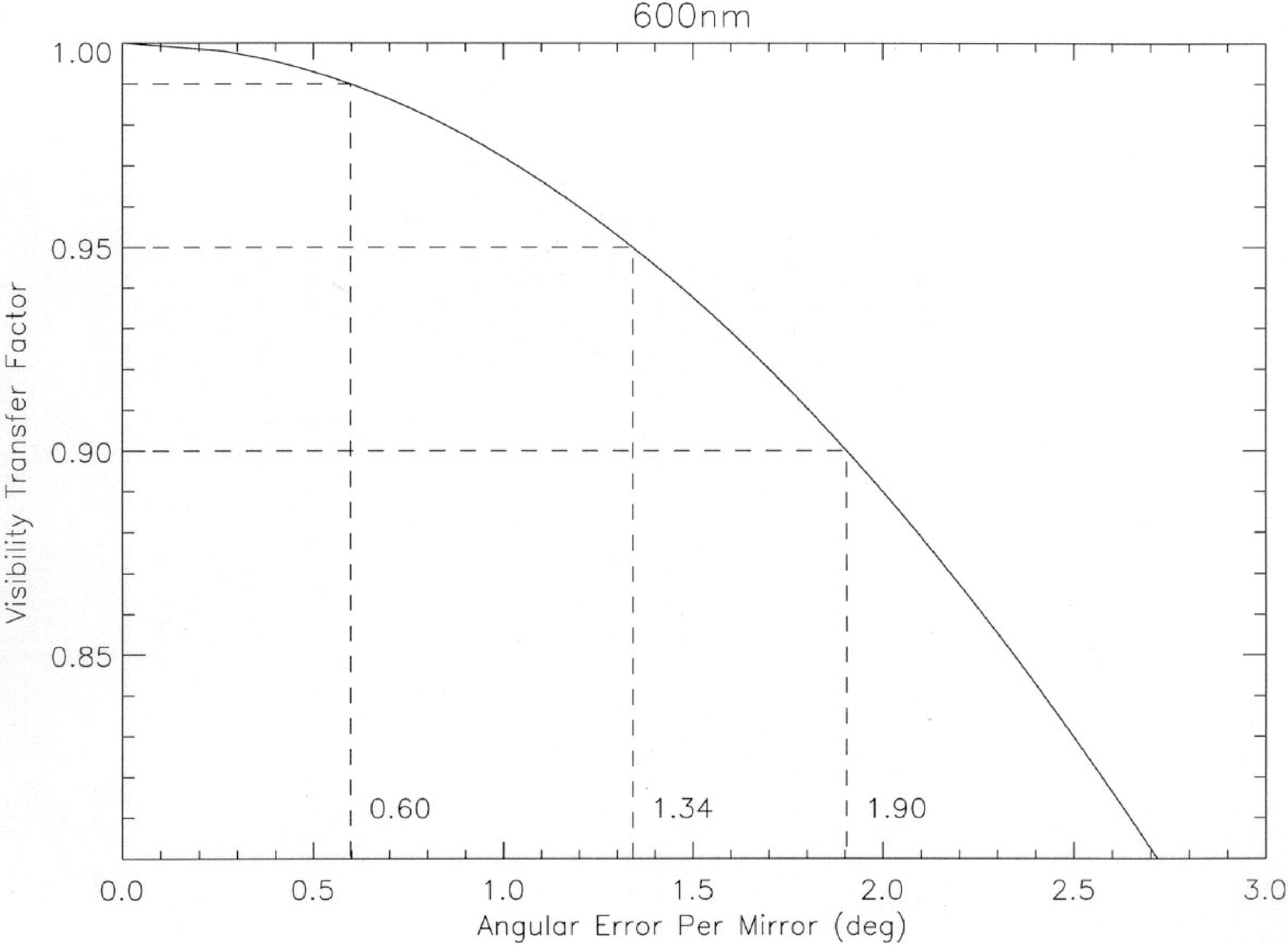
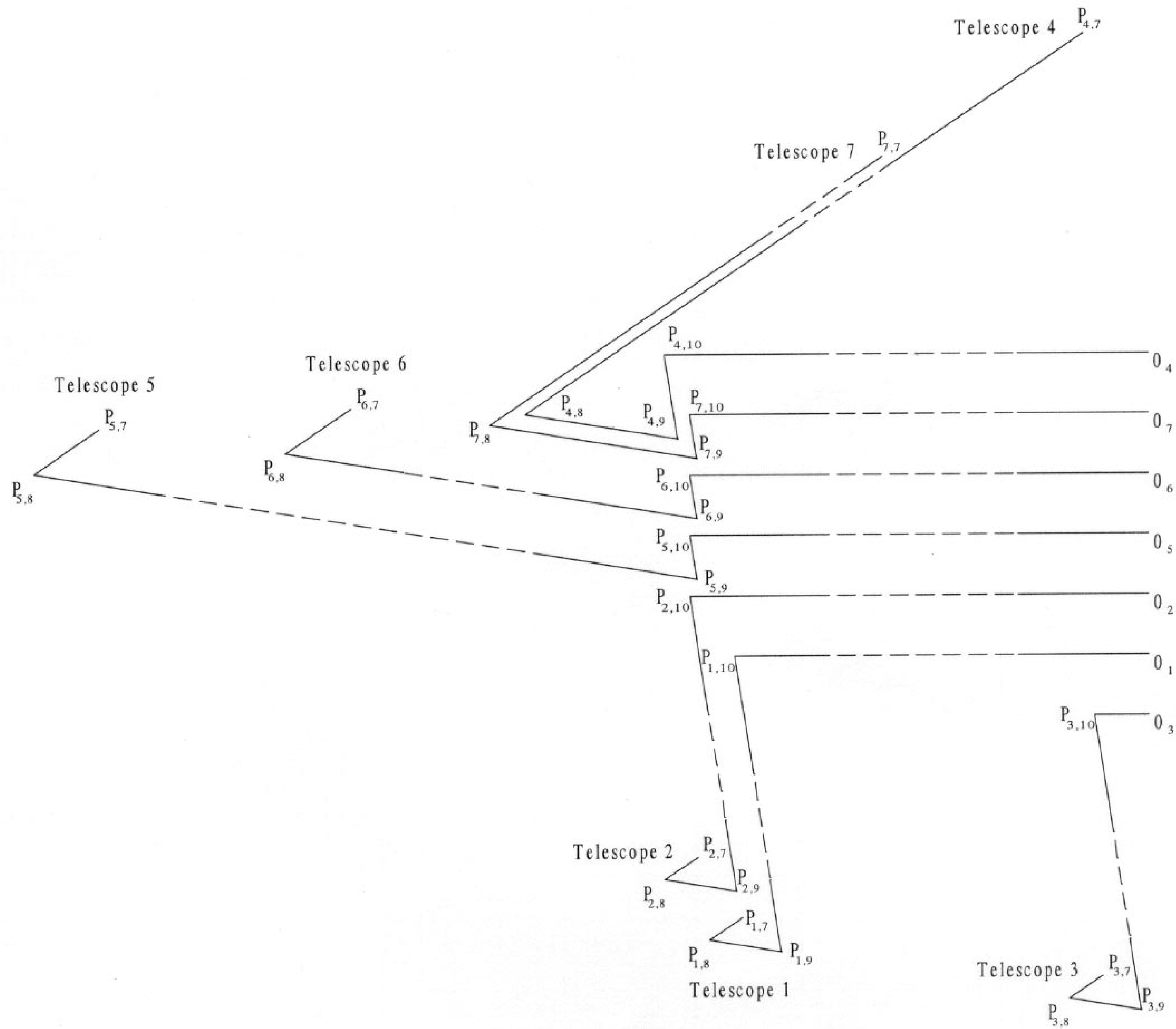


Fig. 2. Calculated s-p phase shift, based on measurements of the complex index of refraction of a silver-over-chrome mirror, at 6 wavelengths.


600nm
Visibility Transfer Factor
1.00
0.95
0.90
0.85
0.60
1.34
1.90
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Angular Error Per Mirror (deg)



Telescope 4
$P_{4,7}$
Telescope 7
$P_{7,7}$
$P_{4,10}$
O_4
Telescope 6
Telescope 5
$P_{6,7}$
$P_{5,7}$
$P_{4,8}$
$P_{4,9}$
$P_{7,10}$
O_7
$P_{7,8}$
$P_{7,9}$
$P_{6,8}$
$P_{5,8}$
$P_{6,10}$
O_6
$P_{6,9}$
$P_{5,10}$
O_5
$P_{5,9}$
$P_{2,10}$
O_2
$P_{1,10}$
O_1
$P_{3,10}$
O_3
Telescope 2
$P_{2,7}$
$P_{2,9}$
$P_{2,8}$
$P_{1,7}$
$P_{1,8}$
$P_{1,9}$
Telescope 1
Telescope 3
$P_{3,7}$
$P_{3,9}$
$P_{3,8}$

Dispersion

- Problems exist even with vacuum delay lines.
- Easy to model, harder to model accurately.
- Makes it more important to have a calibrator close to your source.

- Very much a problem $\bar{c}$ Fibers!
- Correctable $\bar{c}$ glass

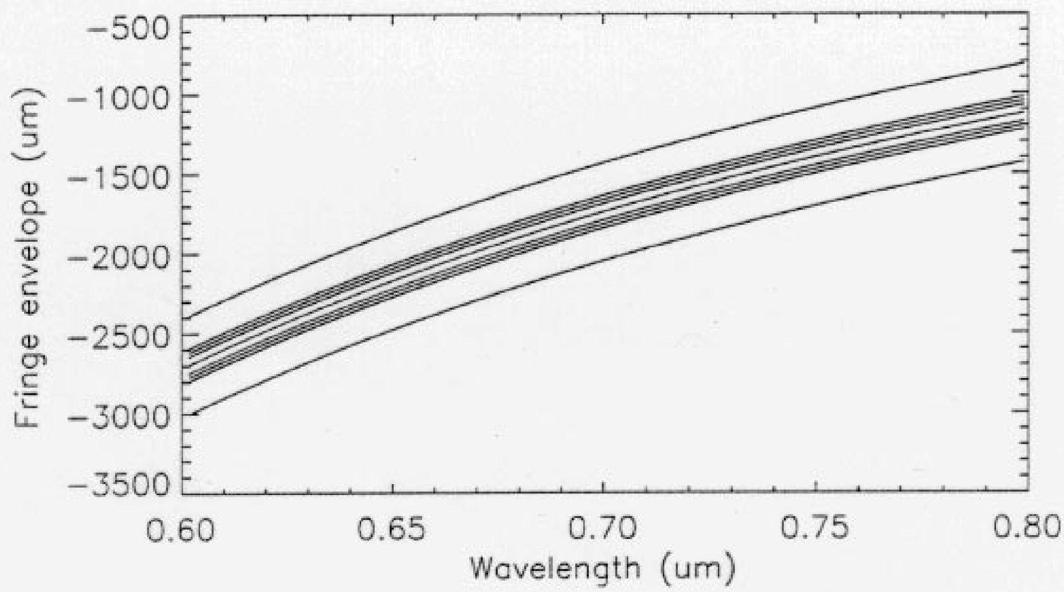


Figure 4: The fringe envelope for a baseline of 350m and a zenith angle of 50 degrees without dispersion correction. The central line represents the center of the envelope, the next pair of lines the 95% point, the next pair the 90% point, the next pair the 85% point and the outermost lines the first zero.

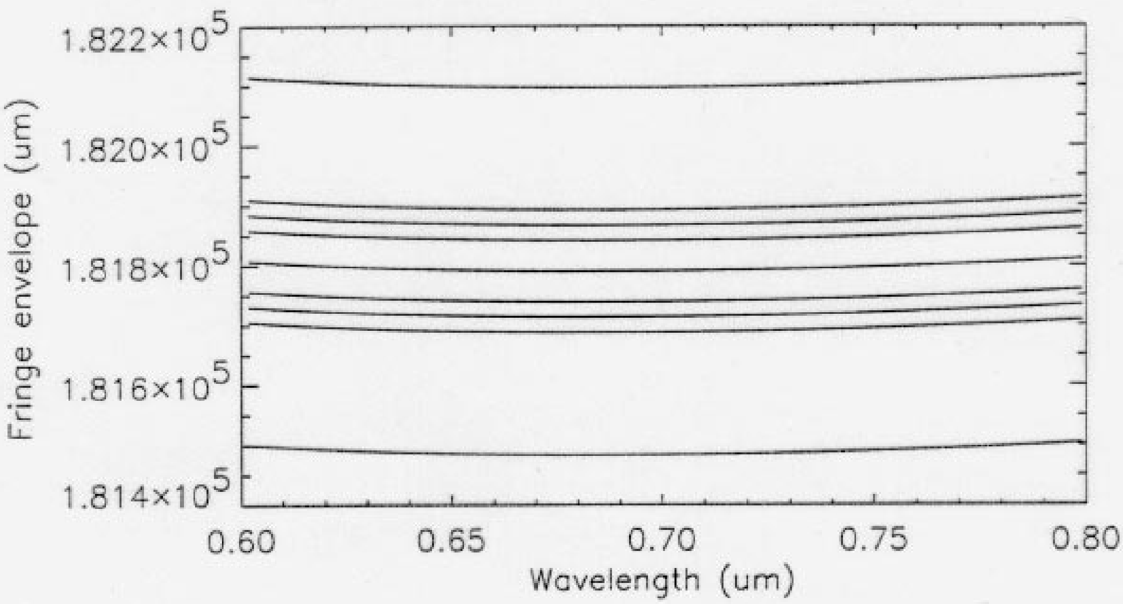


Figure 5: The fringe envelope for a baseline of 350m and a zenith angle of 50 degrees with dispersion correction. The central line represents the center of the envelope, the next pair of lines the 95% point, the next pair the 90% point, the next pair the 85% point and the outermost lines the first zero.

Internal Optical Quality

- Telescope mirror deformations. These are hopefully the same, or similar, in each arm.
- Each optic will introduce a phase variance σ_ϕ^2.
- Do they really add in an RMS way? Are they truly random?
- Total visibility transfer factor is related to total Strehl and so $\eta \approx \exp(-\sigma_\phi^2)$.
- This is something you can (hopefully) measure in order to calibrate it out.

Diffraction

- Long paths $\longrightarrow$ diffraction.
- Differential paths $\longrightarrow$ differential diffraction.
- Beam reducers make the problem worse.
- The combination of atmospheric turbulence and diffraction is not well understood (at least not be me!).

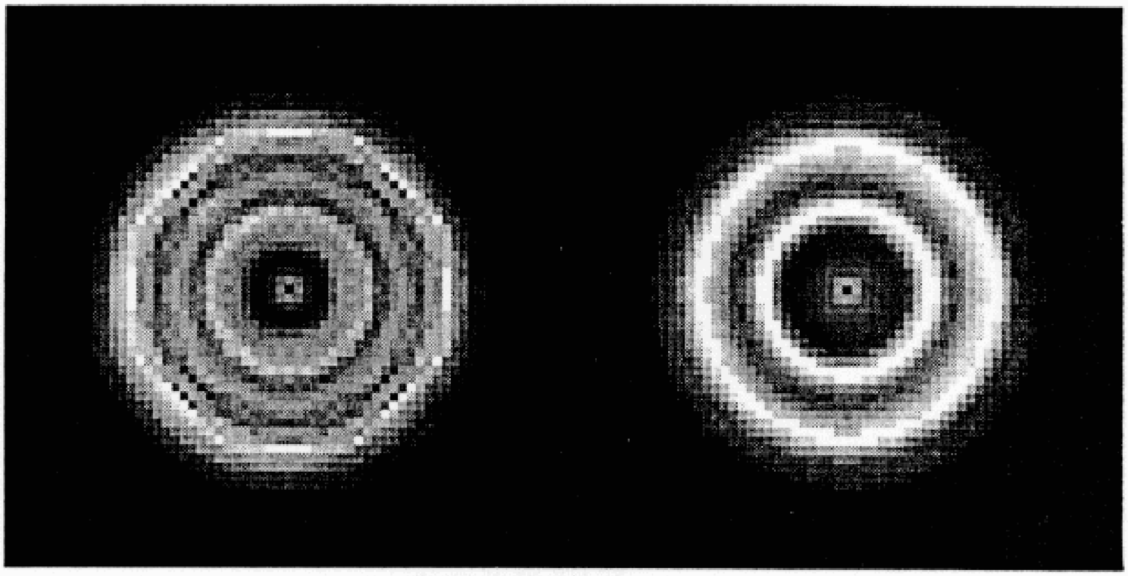


FIGURE E.3. Intensities for beams at 2.2 μm of 1 m diameter reduced to 125 mm and propagated for 100 m (left) and 200 m (right). Beams are apodized with $\Delta n = 8$ cm.

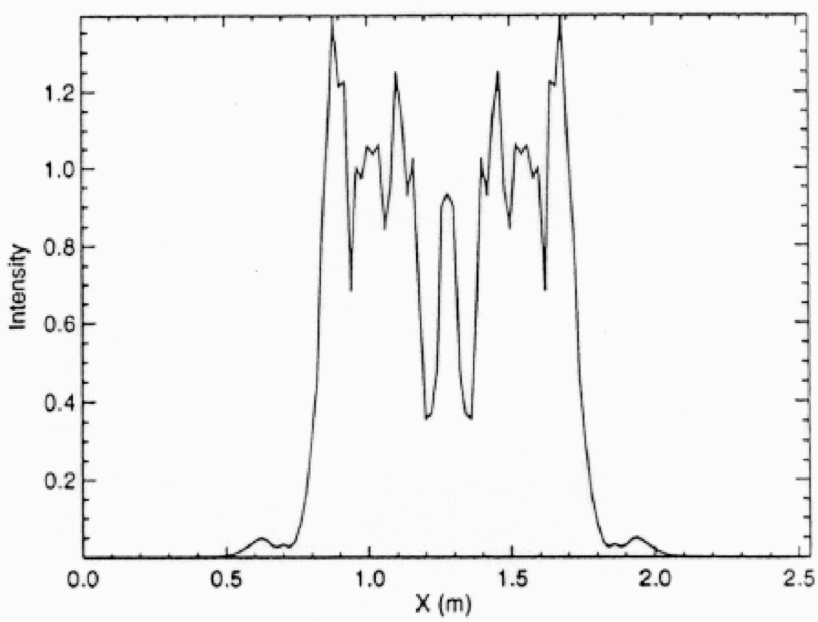


FIGURE E.2. Intensity cross-section for beam at 2.2 μm of 1 m diameter reduced to 125 mm and propagated for 100 m.

The Modern Long-Baseline Stellar Interferometer

- Located at an existing observatory.
- Baselines of up to several hundred meters.
- Probably a N-S baseline initially.
- All beams brought to a central location through evacuated pipes before being directed to the lab.
- Lab building located downwind of telescopes or hidden from view.
- Involves many tons of reinforced concrete.
- Siderostats housed at fixed stations in a partially redundant array.

The Modern Long-Baseline Stellar Interferometer

(continued)

- Siderostats followed immediately by
  1. Beam-reducing telescope (afocal)
  2. Star acquisition camera
  3. Wavefront tip/tilt compensating mirror
- Quadrant detector for tip/tilt system located inside lab, just prior to the beam combiner.
- Delay line is most obvious feature of lab.
- Lab is an air conditioned building-within-a-building for thermal control.
- Delay line trolley includes a momentum compensated piezo and a voice coil actuator.

The Modern Long-Baseline Stellar Interferometer

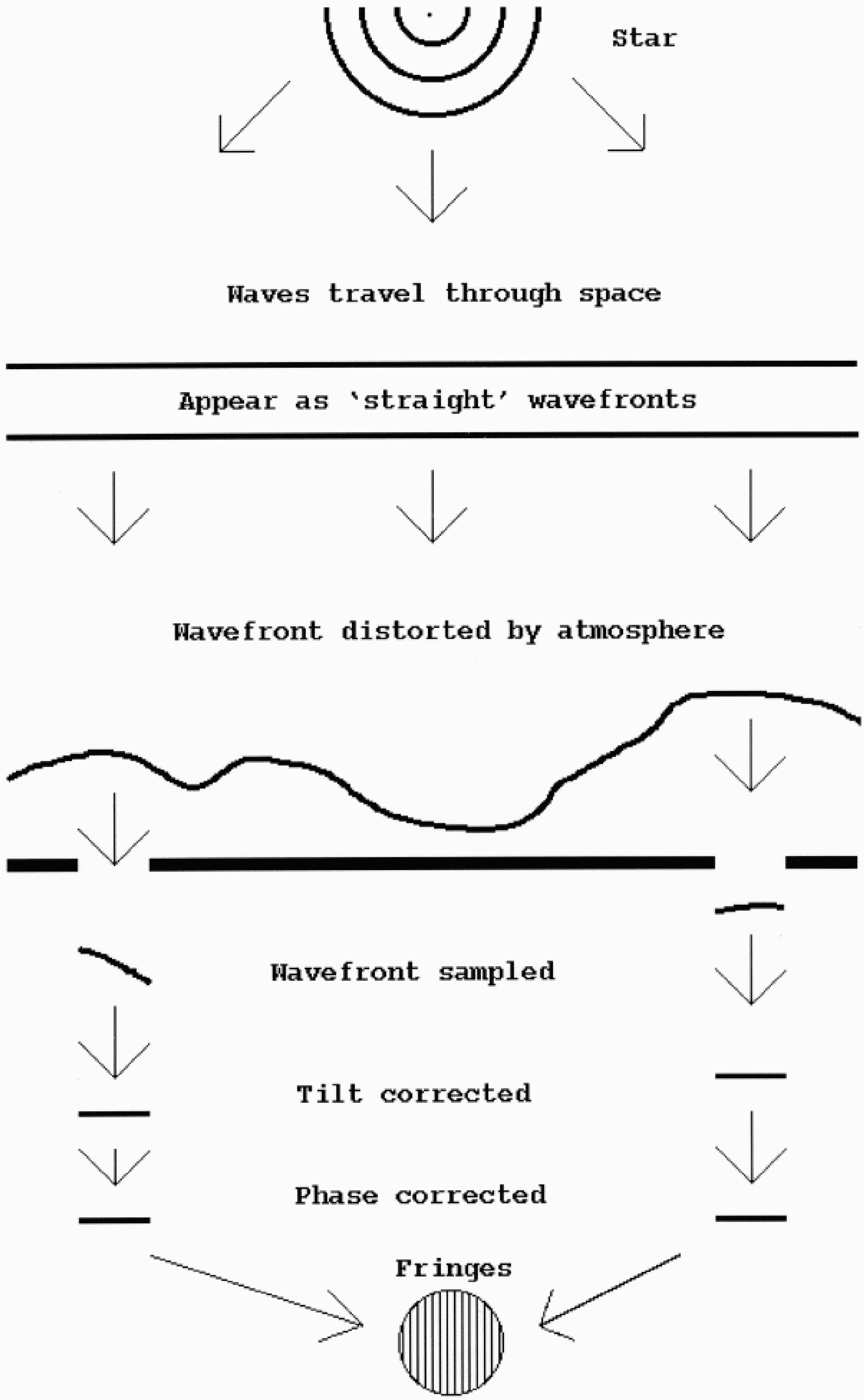
(continued)

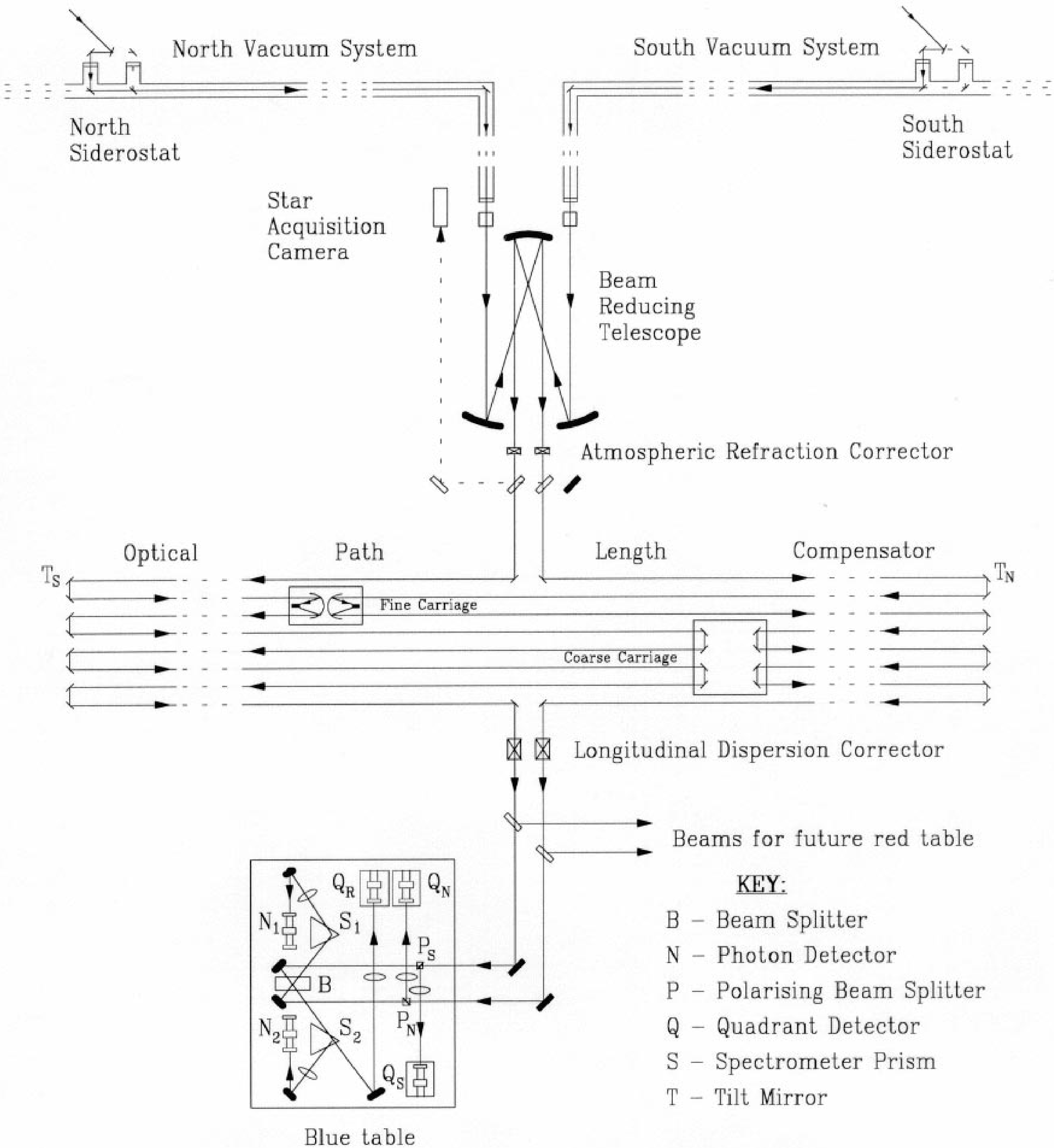
- Main beam-combining table includes:
  1. Quadrant detectors for tip/tilt sensing.
  2. System of mirrors and beam-splitters to combine the beams.
  3. One output beam is used for white-light fringe detection: aperture stop, lens, filter, APD.
  4. One output beam is dispersed in a spectrometer: aperture stop, prism, lens, array detector.
  5. If your wavelength is 2.2 microns you also have spatial filters.

The Modern Long-Baseline Stellar Interferometer

(continued)

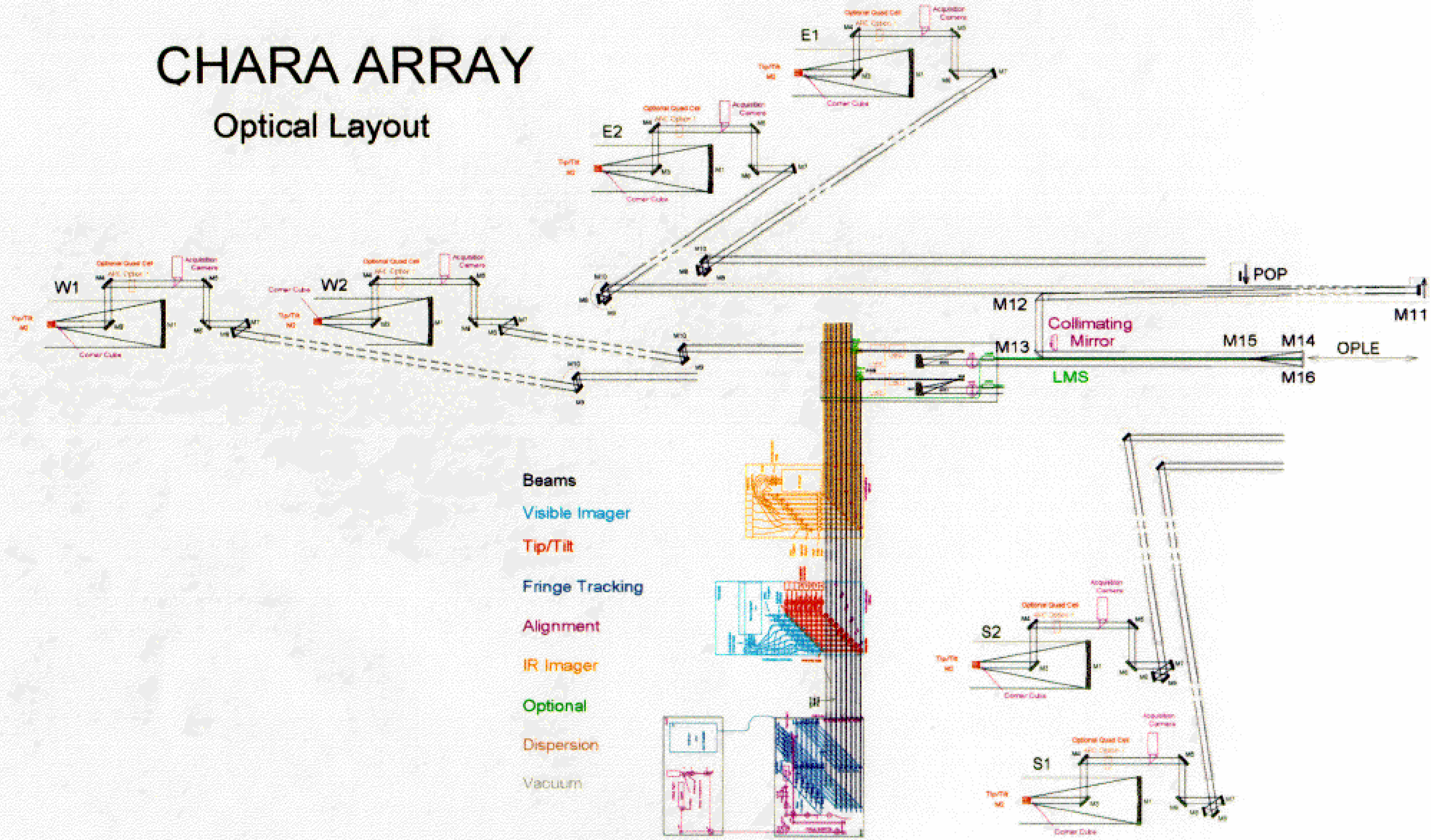
- **Main table also has alignment optics:**
  1. **Pinhole to serve as an alignment fiducial, illuminated by**
     - **HeNe laser used for alignment out to the siderostats.**
     - **Tungsten lamp for an artificial star and channeled spectrum.**
  2. **Theodolite equiped with an illuminated target and a CCD video camera.**
  3. **Numerous corner-cubes for autocollimation.**
- **White paper targets, cardboard, duct tape...**


Star
Waves travel through space
Appear as 'straight' wavefronts
Wavefront distorted by atmosphere
Wavefront sampled
Tilt corrected
Phase corrected
Fringes

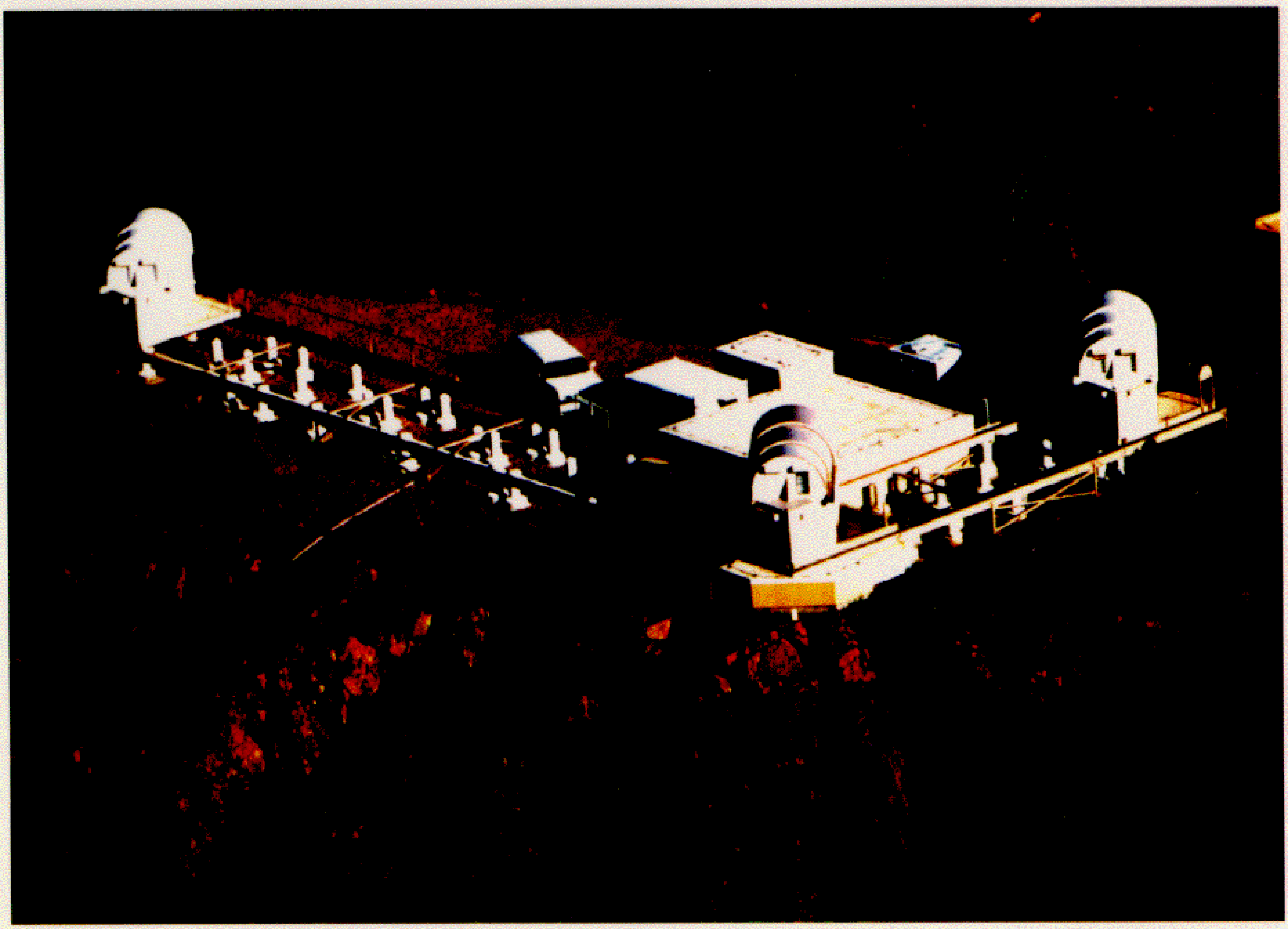

North Vacuum System
South Vacuum System
North Siderostat
South Siderostat
Star Acquisition Camera
Beam Reducing Telescope
Atmospheric Refraction Corrector
Optical Path Length Compensator
T_S
T_N
Fine Carriage
Coarse Carriage
Longitudinal Dispersion Corrector
Beams for future red table
Q_R
Q_N
N_1
S_1
P_S
B
N_2
S_2
P_N
Q_S
Blue table
KEY:
B – Beam Splitter
N – Photon Detector
P – Polarising Beam Splitter
Q – Quadrant Detector
S – Spectrometer Prism
T – Tilt Mirror

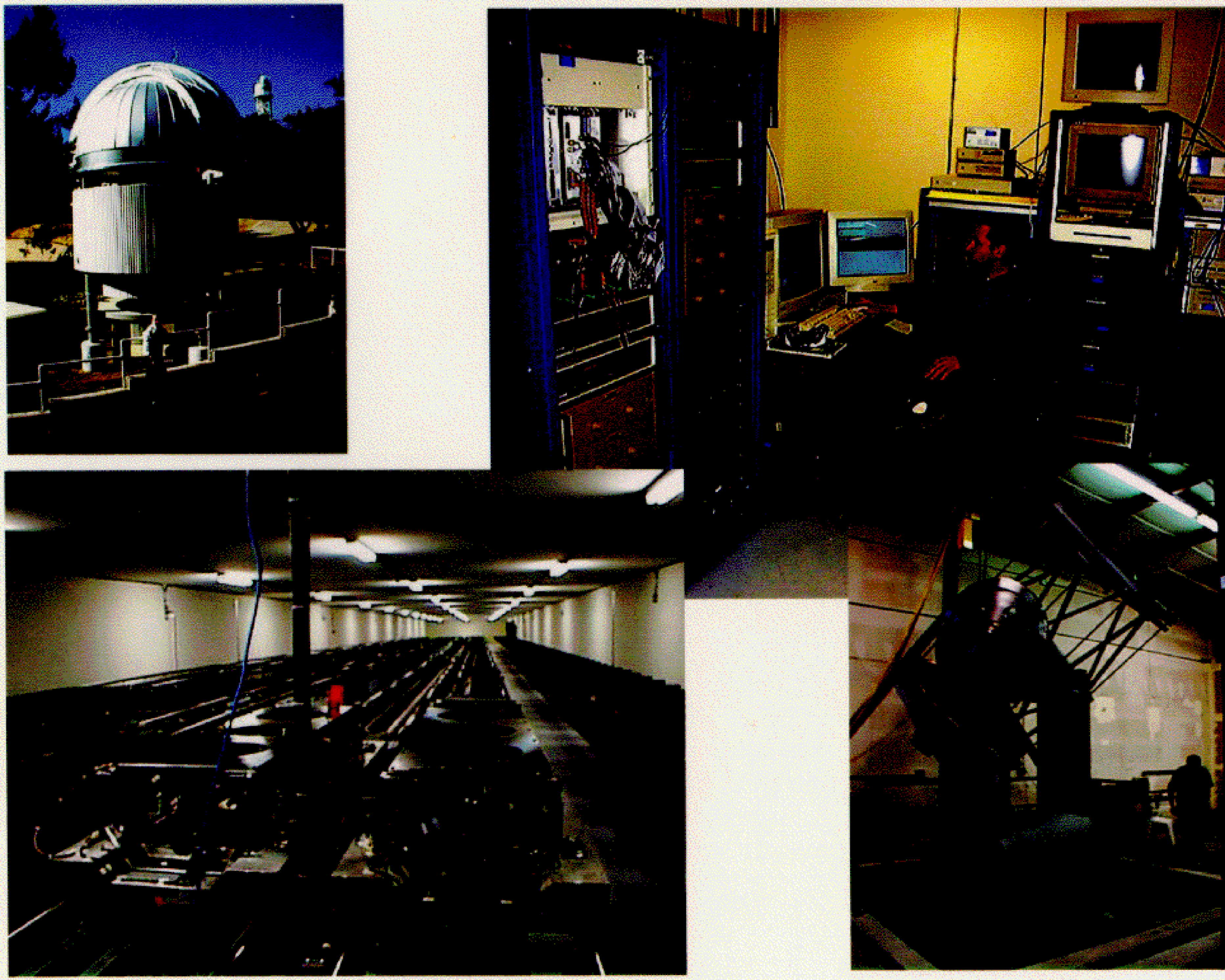
CHARA ARRAY

Optical Layout

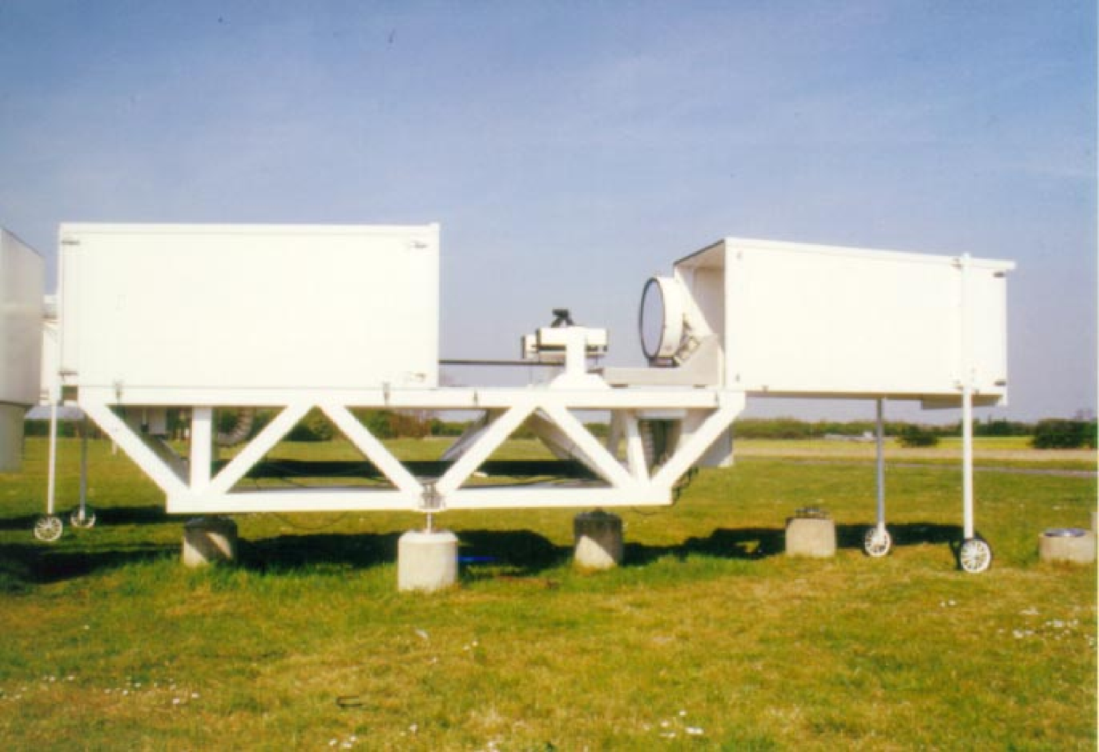


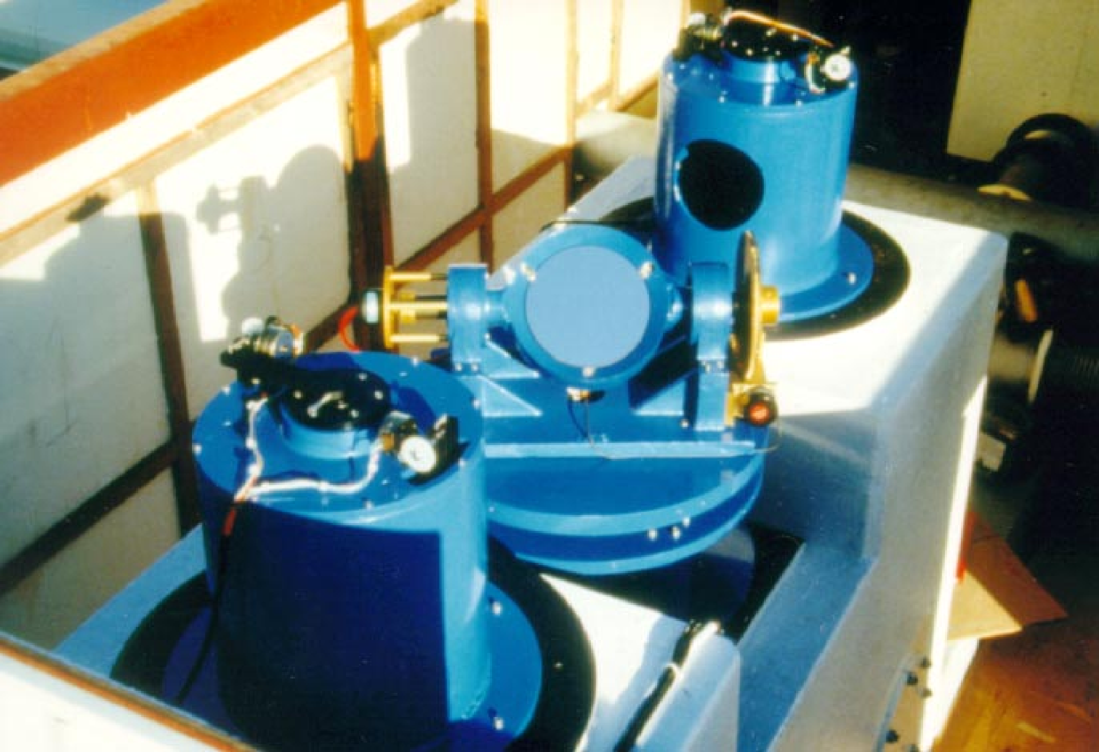
IOTA

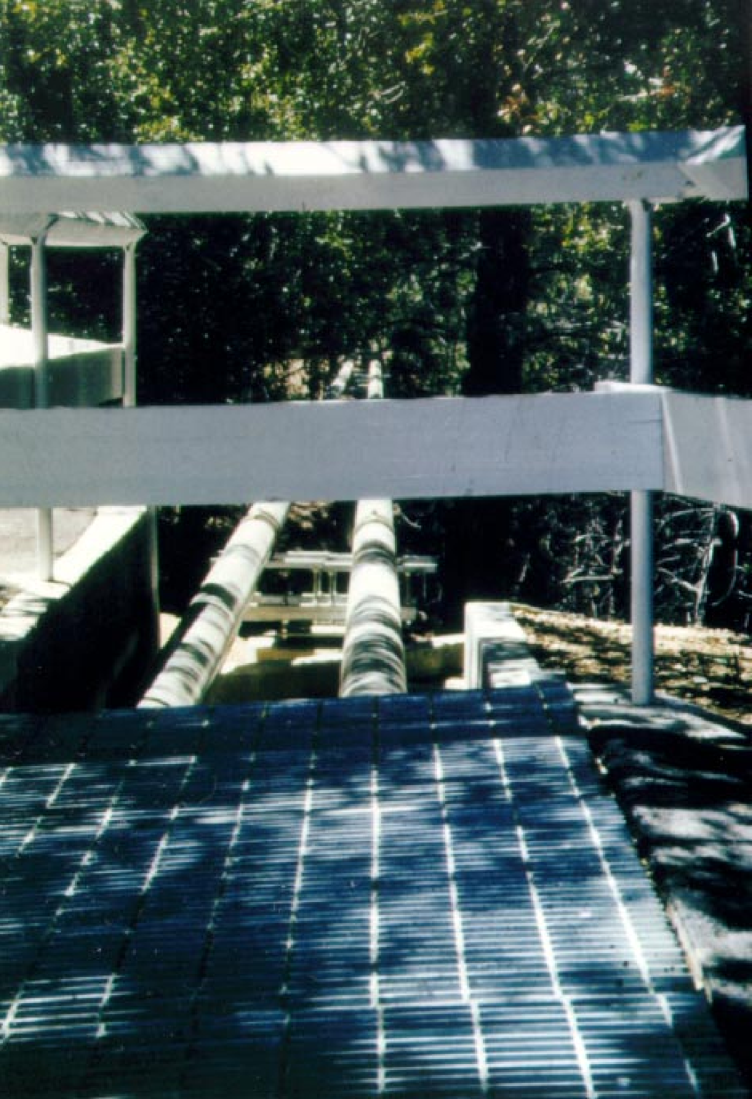


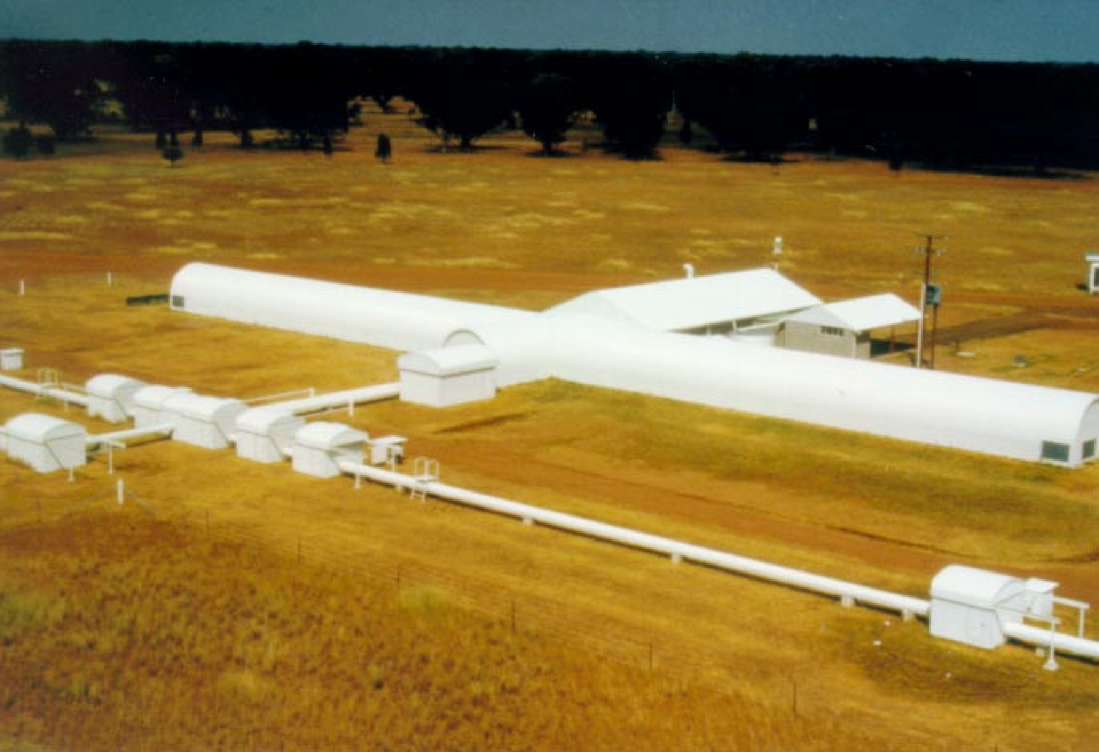


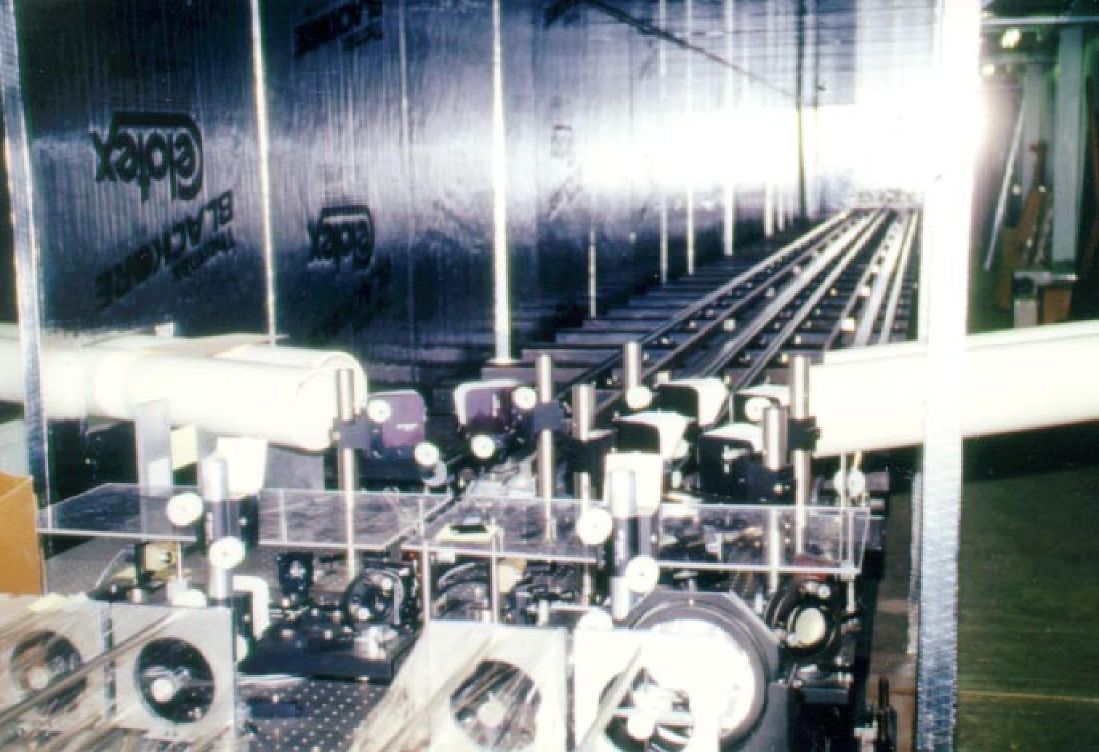

60

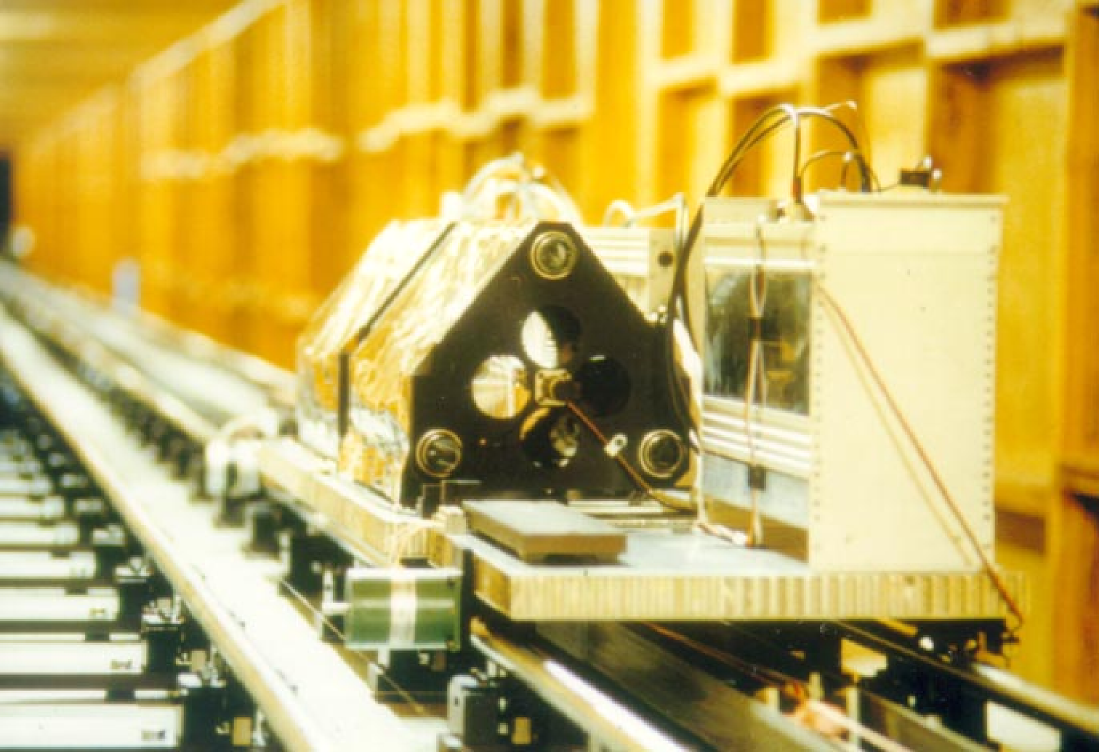


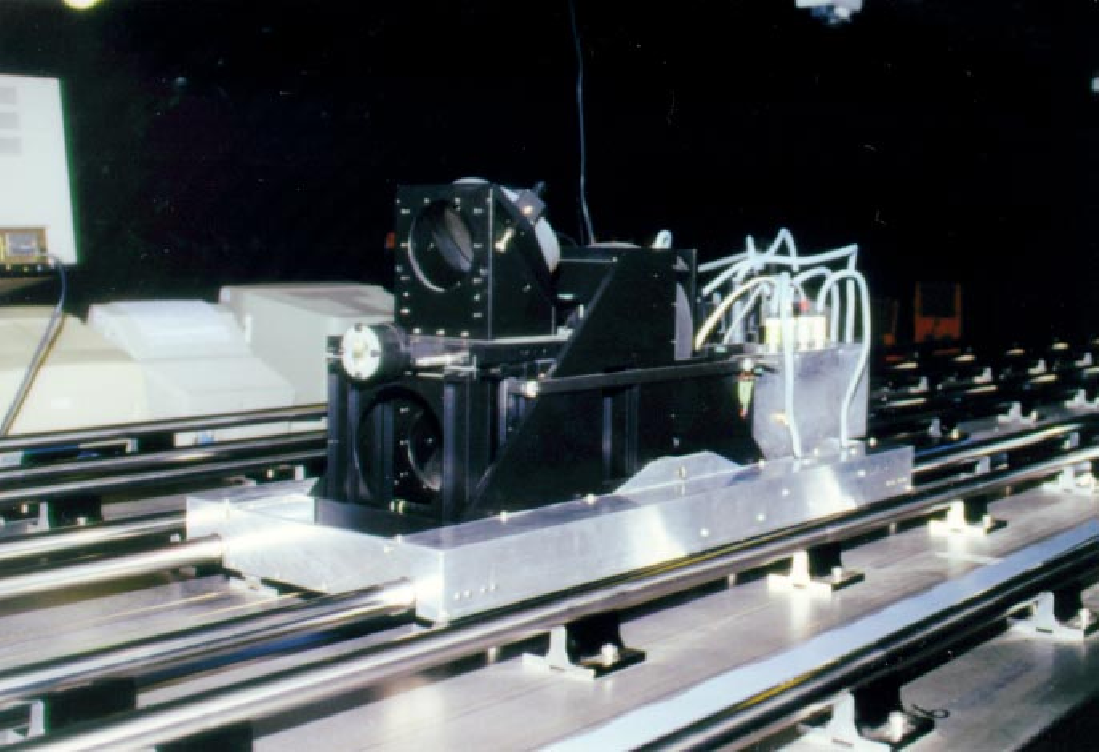


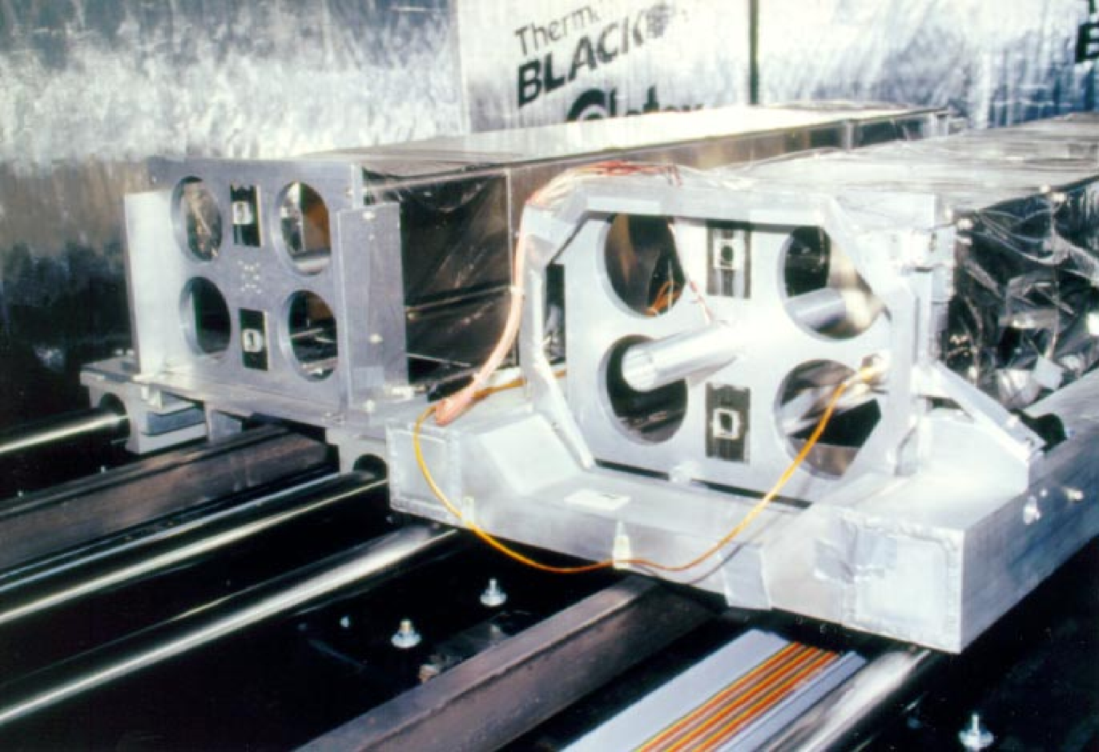











BLACK